ABSTRACTS

Invited Talks

Domain Decomposition and hp-adaptive Finite Elements

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Abstract: We will discuss our on-going investigation of generalizing the parallel h-adaptive method of Bank and Holst to the case of p- and hp-adaptive finite element methods. Besides a posterior error estimation and the adaptive procedures themselves, we will discuss generalization of the DD-solver developed to support the h-adaptive version of the algorithm. At this early point in the investigation there remain many outstanding issues, both mathematical and algorithmic, that will be highlighted.

Heterogeneous Domain Decomposition Methods for Coupled Hydrological Processes

Berninger, Heiko *; Kornhuber, Ralf; Sander, Oliver

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Abstract: The talk focusses on coupled nonlinear problems arising from the simulation of saturated-unsaturated fluid flow in porous media. It introduces an approach for the numerical solution of the Richards equation in heterogeneous soil which completely avoids linearization. This is achieved by applying Kirchhoff’s transformation followed by convex minimization separately in subdomains with homogeneous soil. The coupling is induced by continuity of the pressure and the water flux across the interfaces and solved by the Dirichlet–Neumann or the Robin method. Here, convergence results in 1D as well as systematic numerical experiments in 2D, which show surprising similarities to linear cases, are presented.

Coupling of saturated-unsaturated groundwater flow with surface water is provided by mass conservation and hydrostatic pressure. For surface water reservoirs this coupling can be numerically realized since our solver for the Richards equation also determines the free boundary given by Signorini-type conditions on seepage faces around lakes. In addition, we introduce a solver of Dirichlet–Neumann type for the coupling of the Richards equation with the shallow water equations. Numerical results concerning the 2D-1D coupling are presented.
Better Decisions Through Parallel Software and Hardware Acceleration
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Abstract: For large oil and gas fields, vast amounts of seismic, geologic, and dynamic reservoir data yield high resolution geological models. Due to the limitation of traditional reservoir simulators, high resolution models have been upcaled to field simulation models by reducing the number of gridblock cells from millions to several hundred thousands. The resulting upcaled reservoir simulators often produce poor oil and gas recovery and predict poor field performance.

To fully utilize the available data and enable development and production decision-making in a reasonable time, parallel reservoir simulators have been developed to handle mega-cell (million) to giga-cell (billion) field simulations. These parallel simulations have enabled to recover additional oil and gas because of better understanding of chemical and physical mechanisms involved, process design, and uncertainty analysis.

In this presentation, the speaker will give an overview of the current development of parallel reservoir simulators, recent advances, existing challenges, and future directions. This presentation will also address such important and practical issues as load balance, cell interface treatment, optimum use of multiple cores (CPUs and GPUs), and message passing (MPI and OpenMP) in reservoir simulation applications.

Neumann–Neumann Methods For A DG Discretization of Elliptic Problems with Discontinuous Coefficients on Geometrically Nonconforming Substructures
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Abstract: A discontinuous Galerkin discretization for second order elliptic equations with discontinuous coefficients in 2-D is considered. The domain of interest is assumed to be a union of polygonal substructures \( \Omega_j \) of size \( \Theta(h) \). We allow this substructure decomposition to be geometrically nonconforming. Inside each substructure \( \Omega_j \), a conforming finite element space associated to a triangulation \( \mathcal{T}_h(\Omega_j) \) is introduced. To handle the nonmatching meshes across \( \partial\Omega_j \), a discontinuous Galerkin discretization is considered. In this talk additive and hybrid Neumann–Neumann Schwarz methods are discussed. Under natural assumptions on the coefficients and on the mesh sizes across \( \partial\Omega_j \), a condition number estimate \( C(1 + \max_j \log \frac{H_j}{h_j})^2 \) is established with \( C \) independent of \( h_j, H_j, h_j/H_j \), and the jumps of the coefficients.

The method is well suited for parallel computations and can be straightforwardly extended to three dimensional problems. Numerical results will be included.
Domain Decomposition Methods in Quantum Mechanics

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Abstract: Domain decomposition methods are becoming important tools in electronic structure analysis and quantum dynamics. There are interesting new features for this class of problems. This talk will review the work done in this area and the challenges that remain.

Efficient Adaptive Finite Element Methods

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Abstract: We develop an efficient adaptive finite element methods for the elliptic problems. For the mesh adaption, we combine the mesh refinement and mesh moving techniques based on centroidal voronoi tessellation(CVT) to generate high quality meshes, which in turn provides superconvergent gradients/flux approximation. The recovery type a posteriori error estimators are employed to guide the adaptive mesh generation. A superconvergent cluster recovery method for gradient/flux recovery is presented and it shows asymptotically exactness of the a posteriori error estimator. Finally, we develop a posteriori error estimator by using the simple smoothing iteration of the multigrid method. Plenty of numerical examples illustrate the high efficiency of our adaptive finite element methods.
Discontinuous Galerkin for Space-Time Nonconforming
Optimized Schwarz Waveform Relaxation

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Abstract: In many fields of applications arises the necessity to couple models with very different spatial
and time scales. Among them are far field simulations of underground nuclear waste disposal and ocean-
atmosphere coupling. For such problems, nonconforming spatiotime grids are necessary, in order to match
different physical scales. Moreover, as long time computations are involved, a splitting of the time interval
into windows is necessary, with the possibility to use robust and fast solvers in each time window, and
ultimately to design adaptive solvers. Optimized Schwarz Waveform Relaxation methods provide efficient
solvers for coupling heterogeneous problems: first, they are global in time and thus allow for the use of non
conforming space-time discretization in different subdomains, and secondly, using optimized transmission
conditions, they need a very small number of iterations for computing an accurate solution. In order to have
a high degree of accuracy, in the context of models with discontinuous coefficients, we use a discontinuous
Galerkin method in time. These methods have gained in popularity as they allow for a rigorous analysis,
while keeping a high degree of accuracy, time-stepping approaches, and ultimately lead to adaptive control
of the time step by a posteriori error analysis.

In this presentation, I shall give an overview of the algorithms we use, for advection diffusion problems
with discontinuous coefficients. The mathematical analysis is carried out on the problem semi-discretized
in time. The algorithm is implemented using mortar methods in order to permit non-matching grids in time and
space on the boundary. I will present numerical results in two dimensions which extend the domain of
validity of the approach to the fully discrete problem.

This work has been carried out under collaborations with Professor Laurence Halpern, in Paris 13
University, France, partially supported by French ANR (COMMA) and GdR MoMaS, and Dr. J’erémie
Szeftel, at Princeton University, USA and Bordeaux University, France, partially supported by NSF Grant
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Domain Decomposition Methods for Electromagnetic Wave Propagation Problems
Involving Heterogeneous Media and Complex Domains

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Abstract: Electromagnetic (EM) waves are ubiquitous in present day technology. Indeed, electromagnetism has found and continues to find applications in a wide array of areas, encompassing both industrial and military purposes. Equally notable are societal applications, in particular those concerned with the question of the existence of adverse effects resulting from the interaction of EM waves with humans, or those dealing with medical applications of EM waves. Although the principles of electromagnetics are well understood, solving Maxwell’s equations for the simulation of realistic wave propagation problems is still a challenge. For practical applications, the solution of such problems is complicated by the detailed geometrical features of scattering objects, the physical properties of the propagation medium and the characteristics of the radiating sources. In addition, because the wavelength is often short, the algebraic systems resulting from the discretization of Maxwell’s equations can be extremely large. Domain decomposition principles are thus ideally suited for the design of efficient and scalable solvers for such systems.

In this talk we will discuss about our recent efforts towards the development of domain decomposition methods coupled to discontinuous Galerkin discretization formulations on unstructured meshes for the solution of the system of timeharmonic Maxwell equations, in view of the simulation of electromagnetic wave propagation problems involving heterogeneous media and complex domains. The domain decomposition methods considered in this study are based on Schwarz algorithms with classical or optimized interface conditions. We will briefly review recent results regarding the formulation of optimized interface conditions for the timeharmonic Maxwell equations and discuss in more details their use in the framework of discontinuous Galerkin discretization methods. Numerical results will be presented for some academic two-dimensional problems allowing a detailed assessment of the properties of the resulting domain decomposition solvers, and for challenging three-dimensional problems involving the interaction of electromagnetic waves with complex geometrical models of human organs and tissues.
**Adaptive and Multilevel BDDC in 3D**

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**Abstract:** This talk is based on joint work with Bedřich Sousedík and Jakub Šístek. Adaptive selection of face coarse degrees of freedom in BDDC is shown to result in a robust and efficient algorithm suitable for hard industrial problems. A new, much simpler procedure for selection of the coarse degrees of freedom is developed. To achieve variable number of constraints on the faces between the substructures, we generalize the implementation of BDDC by change of variables. Multilevel extension for very large problems is also considered.

**Domain Decomposition Methods for Elastic Multi-structure Problems**

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**Abstract:** Elastic multi-structures are composed of a number of substructures that have the same or different dimensions (e.g., bodies, plates, beams, etc.), coupled together with certain junction conditions. They are widely used in the fields of aviation, aerospace, civil engineering, mechanical manufacturing, etc. In the past few decades, much work has been done about mathematical modeling, analysis and numerical solutions for these problems. Elastic multi-structures have a significant feature which looks very complicated from a global view point, but the substructures themselves are quite simple comparably. Therefore, these problems are particularly suitable for solutions through nonoverlapping domain decomposition methods (DDM). In this talk, we will discuss a nonoverlapping domain decomposition method for solving general elastic body-plate problems, based on P1-NZT finite element discretization. The method only requires numerical solution of a pure body problem as well as a pure plate problem at each iteration step, which can be implemented by existing efficient numerical solvers. By introducing a Clément-type operator with error estimates and deriving certain spectral equivalence lemma, we can demonstrate that the convergence rate of the method is optimal which is independent of the mesh size even for a shape-regular finite element triangulation. This enables us to combine the method with adaptive techniques in practical applications. Our convergence analysis might be helpful in investigating convergence rates of some other nonoverlapping DDM based on any shape-regular finite element triangulations.
The Role of Fast Solvers in Modeling and Simulation of Biological Systems

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Abstract: A typical feature of biological systems is their high complexity and variability. This makes modelling and computation very difficult, in particular for detailed models based on first principles. The problem starts with modelling geometry, which has to extract the essential features from those highly complex and variable phenotypes and at the same time has to take in to account the stochastic variability. Moreover, models of the highly complex processes which are going on these geometries are far from being well established, since those are highly complex too and often couple on a hierarchy of scales in space and time. Simulating such systems always puts the whole approach to test, including modeling, numerical methods and software implementations. In combination with validation based on experimental data, all components have to be enhanced to reach a reliable solving strategy.

To handle problems of this complexity, new mathematical methods and software tools are required. In recent years, new approaches such as Filtering Algebraic Multigrid methods and corresponding software tools have been developed allowing to treat problems of huge complexity. In the lecture we report on the numerical simulation of the diffusion of xenobiotics through human skin. First computations for this problem were made in the last decade, yielding new insight into permeation pathways through human skin, which were confirmed experimentally ten years later.

Localization Based Finite Element Electronic Structure Calculations

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Abstract: In this presentation, we will talk about localization based finite element approximations in electronic structure calculations. We will report several numerical experiments in materials science, which show that our localization based finite element approach is efficient. This presentation is based on some joint works with X. Dai, X. Gong, L. Shen, and D. Zhang.
Minisymposium Talks

1. Continuous and Discrete Optimized Schwarz Methods
Organisers: Gander, Martin

Abstract: Optimized Schwarz methods are iterative domain decomposition methods which use more effective transmission conditions between subdomains instead of the classical Dirichlet conditions. Most analysis of optimized Schwarz methods is based on Fourier techniques, and is thus restricted to simple domain decompositions and constant coefficient operators. This minisymposium has the purpose of reuniting researchers which try to go beyond these restrictions with optimized Schwarz methods. There will be new convergence results for optimized Schwarz methods with curved interfaces, where Fourier analysis is not applicable, algebraically optimized transmission conditions, a new approach for high order transmission conditions, an analysis which shows that the discretization of a continuous Schwarz method can actually make it converge faster, and finally an optimal, algebraically derived transmission condition which leads to convergence in two iterations independently of the topology and the number of subdomains used.

Can the Discretization Speed up Schwarz Methods?
Dolean, Victorita; Gander, Martin J.

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Abstract: Schwarz methods can be analyzed both at the continuous and discrete level. While convergence results at the discrete level are valid for a particular discretization only, convergence results at the continuous level are more general, and often remain valid for suitable discretizations.

The goal of our presentation is to show that in some cases the discretization can have an influence on the convergence behavior of Schwarz methods. We consider two model problems: Laplace's equation and the Cauchy Riemann equations. We first show that for Laplace's equation the continuous convergence analysis remains valid for discretizations. For the Cauchy Riemann equations, we prove that the discretized algorithm converges faster than the continuous convergence analysis predicts. We explain this by showing that the discretization introduces an additional term in the interface condition, of the form used in optimized Schwarz methods, which leads to the faster convergence observed.
Absorbing Boundary Conditions and Perfectly Matched Layers:  
a Mathematical Equivalence  
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Abstract: In order to truncate infinite computational domains, there are two major competitors: absorbing boundary conditions and perfectly matched layers. The former use approximations of the Dirichlet to Neumann map, and the latter adds a layer outside the truncated domain, in which a modified equation is solved, which absorbs outgoing parts of the solution.

Using the concept of the pole condition, we show for a model problem that these two seemingly very different techniques are in fact related. For the particular case of an absorbing boundary condition given by a Padé approximation of the Dirichlet to Neumann map, we show that a natural implementation leads to a layer structure corresponding to a perfectly matched layer with an exponentially scaled outer problem.

Optimal Interface Conditions for an Arbitrary Decomposition into Subdomains  
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Abstract: Recent advances in the design of optimized transmission conditions in domain decomposition methods (DDMs) have improved the convergence rate of such methods significantly. These efficient transmission conditions are usually local approximations of the nonlocal Dirichlet-to-Neumann operators, which are known to lead to DDMs that converge in a finite number of iterations. Unfortunately, operators leading to finite convergence may not exist in the many-subdomain case; even when they do exist, they must be derived separately for each PDE. In this talk, we present an algebraic, Schur complement based approach for systematically deriving nonlocal boundary operators that lead to convergence in finitely many steps for arbitrary decompositions (including those that contain cross points), as long as the subdomain problems are well posed and the subdomains are connected. We will also comment on how these operators can be approximated cheaply by solving recurrence relations.
An Algebraically Optimized Schwarz Method
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Abstract: Optimized Schwarz Methods (OSM) use Robin transmission conditions across the subdomain interfaces. The Robin parameter can then be optimized to obtain the fastest convergence. A new formulation is presented with a coarse grid correction. The optimal parameter is computed for a model problem on a cylinder, together with the corresponding convergence factor which is smaller than that of classical Schwarz methods. Numerical experiments illustrating the effectiveness of OSM with a coarse grid correction, both as an iteration and as a preconditioner, are presented.

An Optimized Schwarz Method for Domains with an Arbitrary Interface
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Abstract: Up to now, optimized Schwarz methods have been studied using Fourier analysis, so results only hold for regular geometry. For non-overlapping domain decomposition, we use Poincare-Steklov operators to obtain convergence results for arbitrary domains for first-order as well as certain higher-order artificial boundary conditions. For first-order artificial boundary condition, an upper bound of the spectral radius of the relevant fixed point operator is $1+O(h^{1/2})$ where $h$ is the discretization parameter while for a higher-order boundary condition, a spectral radius estimate is $1+O(h^{1/4})$. These results agree with the convergence rates available in the literature for rectangular domains.
2. Schwarz Methods for Multiphysics Problems on Large Scale Parallel Computers

Organizers: Cai, Xiaochuan; Yang, Chao

Abstract: Based on the classical Schwarz alternating algorithm, a variant of overlapping Schwarz algorithms has been developed in recent years for the purpose for solving large scale multiphysics problems on computers with large number of processors. In this mini-symposium, we present some successful applications of the algorithms in several rather difficult multiphysics problems including global climate modeling, modeling of fluid mixing in micro-channels, modeling of plasma flows, modeling of blood flow in human arteries, and some inverse elastic material problems.

Two-level Methods for Simulation of Blood Flows in Arteries

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(Joint work with Barker, Andrew)

Abstract: Simulation of fluid-structure interaction is a complex problem that involves modeling different physics for the fluid and the structure and coupling them together in a stable and efficient manner. In this talk we discuss scalable techniques in the multilevel Newton-Krylov-Schwarz family for solving the nonlinear, monolithically coupled fluid-structure interaction system on moving finite element meshes in the arbitrary Lagrangian-Eulerian framework. We report numerical results obtained on supercomputers for the simulation of blood flows in arteries.
A Parallel Additive Schwarz Preconditioned Jacobi-Davidson Algorithm for Polynomial Eigenvalue Problems in Quantum Dot Simulation

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Abstract: We shall introduce a newly developed parallel scientific software package, called the parallel Jacobi-Davidson (PJDPack) package, using the PETSc and the SLEPc for finding a few eigenvalues of polynomial eigenvalue problems (PEPs). Our target applications include cubic and quintic PEPs arising in semiconductor quantum dot simulations. PJDPack is implemented based mainly on Jacobi-Davidson (JD) algorithm, which belongs to a class of subspace methods. The JD algorithm consists of two key steps. One first extracts an approximate eigenpair from a given search space using the Rayleigh-Ritz procedure. If the approximate eigenpair is not close enough, one then needs to enlarge the search space by adding a new basis vector, which is the approximate solution of the correction equation. Solving the correction equation is considered to be a key ingredient for the successful convergence of the algorithm. Therefore, to design an efficient preconditioner for the JD algorithm becomes very crucial. Hence we propose a parallel preconditioner based on the Schwarz framework, which is wildly used and well-understood for solving linear systems, but less studied for solving eigenvalue problems. From our numerical experiments, the PJDPack can find all five interior eigenpairs of a quintic polynomial eigenvalue problem with more than 32 million variables within 12 minutes by using 272 processors and can achieve a satisfactory superlinear speedup performance up to 320 processors. Algorithms for both a straight artery model and an end-to-side graft model.

Comparison of Two Fully Implicit Strategies for a Nonlinear MHD System: Newton-Krylov-Schwarz and Nonlinear Multigrid with AMR.

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Abstract: A Schwarz method, working together with a Newton-Krylov method, is a demonstrably effective method for rather general elliptically or parabolically dominated PDE systems, in particular, MHD. However, when nonlinearity and near-singularity are severe, it may stagnate. As an alternative, nonlinear multigrid with Schwarz and/or Newton-Krylov method inside may provide a more robust and efficient method. We consider a model 4-component resistive MHD problem, and compare conventional Newton-Krylov-Schwarz method and nonlinear multigrid method. Newton-Krylov-Schwarz method may be nested as an inner or bottom solver. This approach allows to use existing solvers for individual subsystem (on different grids if necessary). For problems that develop field singularities, an adaptive mesh refinement can be employed which is a natural extension to nonlinear Multigrid and where subsystem are associated with subgrids introduced to deal with local singularities. With toolboxes from PETSc and CHOMBO, we present work in progress showing relative advantages and disadvantages of these methods.
A One-shot Algorithm for Recovering the Lame Parameter in Elastic Materials

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Abstract: Tissue stiffness is a qualitative property to distinguish abnormal tissue from normal tissue, and the stiffness changes are generally described in terms of the Lame coefficient. In this talk, a one-shot Lagrange-Newton-Krylov-Schwarz algorithm is presented to solve inverse elliptic problems of recovering the Lame parameter in elastic materials. The proposed algorithm employs a class of two-level domain decomposition preconditioners in the inexact Newton method. Particular attention is paid to the coarse-level solver strategies, as well as the subdomain solvers. Numerical experiments are presented to show the efficiency and scalability of the algorithm on a machine with up to 1024 processors.

Fully Implicit Domain Decomposition Methods for a Global Shallow Water Model with Topography

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Abstract: A scalable fully implicit solver for the shallow water equations on the cubed-sphere mesh is developed. We use the inexact Newton method to solve the large sparse nonlinear algebraic system resulting from the fully implicit scheme. A Schwarz preconditioned Krylov iterative method is used to solve the Jacobian linear system at every Newton step. When a nonsmooth bottom topography such as mountain is involved, our scheme is incorporated to the Osher’s Riemann solver to satisfy the exact-C property thus spurious oscillations near the nonsmooth area can be avoided. When solving the nonlinear system the nonsmoothness of the mountain leads to another difficulty that some entries of the Jacobian matrix become multiple-valued. A smoothing method is used in our solver to relieve this difficulty. Numerical results obtained on machines with thousands of processor cores are provided.
3. Coarse Discretization Spaces by Integral Constraints and Energy Minimization

Title: "Coarse Spaces for Multiscale Heterogeneous Problems"

Organisers: Scheichl, Robert; Vassilevski, Panayot; Widlund, Olof

Abstract: This mini-symposium aims at bringing together experts in three areas of numerical PDEs that exploit common tools to deal with large scale heterogeneous problems arising in applications, where an appropriate coarse space is indispensable. While coarse spaces are the central ingredient in multigrid methods and are crucial in domain decomposition methods to render the method independent of the number of subdomains, in numerical upscaling the coarse space and its approximation properties are actually the ultimate goal. That is, in numerical upscaling the coarse space is used to produce a discrete representation of the PDE to be used as a more efficient and accurate enough approximation of a typically computationally infeasible fine-grid solution. In all three areas the central problem of current interest is how to choose these coarse spaces such that the convergence of the resulting method is not polluted by fine scale variation.

Domain Decomposition Methods for Electronic Structure Analysis

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Abstract: This talk gives an overview of domain decomposition methods for electronic structure analysis. I will first discuss the case when there is a spectral gap, i.e. the case of insulators. I will then discuss what happens when there is no spectral gap, i.e. the case of metals.

Domain Decomposition Methods for High-contrast Multiscale Problems

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Abstract: In this talk, I will discuss special coarse spaces for multiscale and domain decomposition methods. The focus will be on problems that have high variations in the media properties. The coarse spaces are constructed based on an eigenvalue problem motivated by weighted Poincare inequality. We show that if domain decomposition methods use the proposed coarse spaces then the condition number of preconditioned system is independent of the contrast in media properties. The theoretical results about the independence of the condition number are for problems where there is a separation of the scales in the values of the conductivity. The coarse space can have large dimension. In this talk, we discuss dimension reduction for the proposed coarse spaces. The latter results to coarse spaces that can provide an accurate representation of the solution on a coarse grid. Numerical results will be presented. This is a joint work with Juan Galvis.
Mixed Multiscale Finite Element Methods Using Approximate Global Information
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Abstract: We propose a framework of mixed MsFEMs using approximate global information. The mixed MsFEMs are able to capture the non-local features of the media and improve accuracy significantly compared to local mixed MsFEMs. The mixed MsFEMs can work efficiently for the problems with non-separable hierarchical scales. Analysis is given for the proposed methods. We show the applications of the mixed MsFEMs to deterministic porous media and stochastic porous media.

Coarse Spaces over the Ages
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Abstract: This talk will survey the development of coarse spaces in domain decomposition and iterative substructuring methods for elliptic problems. Special attention will be paid to the theoretical foundations that guarantee scalability of the resulting method. An attempt will be made to present the various coarse spaces from a unified perspective.

Analysis of FETI Methods for Multiscale Elliptic PDEs
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Abstract: In this talk we discuss the conditioning of finite element tearing and interconnecting (FETI) methods for heterogeneous media. We consider the scalar elliptic equation in a two- or three-dimensional domain with a highly heterogeneous (multiscale) diffusion coefficient. This coefficient is allowed to have large jumps not only across but also along subdomain interfaces and in the interior of the subdomains. In other words, the subdomain partitioning does not need to resolve any jumps in the coefficient. Under suitable assumptions, we can show rigorously that the condition numbers of the one-level and the all-oating FETI system are robust with respect to strong variations in the contrast in the coefficient. We get only a dependence on some geometric parameters associated with the coefficient variation. In particular, we can show robustness for so-called face, edge, and vertex islands in high-contrast media. As a central tool we prove and use new weighted Poincare and discrete Sobolev type inequalities that are explicit in the weight. Our theoretical findings are confirmed in a series of numerical experiments.

This work is supported by the Austrian Science Fund (FWF) within the Doctoral Program on Computational Mathematics W1214.
Boundary Layer Technical Tools for Substructuring Methods
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Abstract: In this talk we discuss our recent results on (inexact) substructuring type methods and multiscale PDEs.

Two-level Schwarz and Upscaling
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Abstract: In this talk we draw up links between multilevel iterative solvers and certain upscaling/multiscale techniques for elliptic PDEs with highly variable coefficients. These arise in practice, for example, in the computation of flow in heterogeneous porous media, in both the deterministic and (Monte-Carlo simulated) stochastic cases. When there is no a priori scale separation, standard multiscale techniques require the solution of local "cell" problems in each cell, leading to a computational complexity that can be no better than linear in N, where N is the number of unknowns on the subgrid (globally). Moreover, except for the periodic case, no theory is yet available that analyses the dependency of the accuracy of the upscaled solutions on the coefficient variation. Multilevel iterative methods, such as multigrid or domain decomposition, on the other hand, lead to a similar computational cost with respect to subgrid size (i.e. O(N)), but here the computational cost will in general depend on the coefficient variation. In a series of recent papers we have analysed, both numerically and theoretically, various simple variants of multigrid and domain decomposition methods that are robust to strong coefficient variation. Their similarity to various multiscale techniques provides guidelines for the design of robust multiscale techniques and theoretical tools for their analysis.

This is joint work with I.G. Graham, J. Van lent (both Bath), P. Vassilevski (LLNL) and L. Zikatanov (Penn State)."

Operator-Dependent Approximation Spaces by Constrained Energy Minimization
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Abstract: We consider an unified approach of constructing operator-dependent approximation spaces on relatively coarse computationally feasible meshes. The approach utilizes natural energy functionals associated with the PDEs of interest. We construct local basis functions by minimizing the underlined functional subject to a set of constraints. The constraints are chosen so that the resulting spaces span any a priori given set of functions. We investigate the resulting spaces as upsampling discretization tool in the case of elliptic PDEs (in both standard Galerkin and mixed form), as well as for some hyperbolic equations. In the latter case the approach we consider utilizes nonlinear minimization functionals and adaptive mesh refinement. Some of the resulting discretization methods are illustrated with numerical examples.
**A New Result on Almost Incompressible Elasticity, FETI-DP, and BDDC**

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**Abstract:** Almost incompressible elasticity problems lead to very ill-conditioned linear systems of algebraic equations. They also require extra care in choosing the finite element discretization in order to avoid locking.

Mixed finite element approximations using discontinuous pressure approximations are considered. By eliminating the pressure variables on the element level, symmetric, positive definite, and very ill-conditioned matrices are obtained. The condition number approaches infinity as the Poisson ratio approaches 1/2.

A FETI-DP or BDDC algorithm is characterized by the choice of primal constraints. Once this set of constraints have been chosen, the convergence rates of the two methods is almost identical. The key to the development of a successful algorithm is to choose a good set of primal constraints. As always, the three-dimensional case is the more interesting one and our work is focused exclusively on it.

We have been able to show that a choice of primal constraints that guarantees good performance for the compressible case will work well in the almost incompressible case as well, if we constrain the integrals of the normal displacement over each face to have a common value.

We note that previous studies of these problems have lead to considerably more complicated and quite rich sets of primal constraints; see work by Clark Dohrmann and Jing Li and OW.

This is joint work with Axel Klawonn and Oliver Rheinbach of the University of Essen, Germany.

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**Robust Multilevel Preconditioners for Elliptic Equations with Jump Coefficients on Bisection Grids**

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**Abstract:** In this talk, we are going to design nearly optimal multilevel preconditioners for the finite element approximation of symmetric elliptic problems with jump coefficients on bisection grids. We analyze the eigenvalue distribution of BPX and multigrid preconditioners, and prove that there are only a fixed number of eigenvalues of the preconditioned system which are deteriorated by the large jump. The remaining eigenvalues are bounded nearly uniform with respect to the coefficients and the meshsize. Therefore, the resulting preconditioned conjugate gradient algorithm will converge with nearly optimal rate.
4. Heterogeneous Domain Decomposition

Organizers: Kornhuber, Ralf; Quarteroni, Alfio; Xu, Jinchao

Abstract: Coupled heterogeneous phenomena are not an exception but the rule in advanced numerical simulations of fluid dynamics, hydrodynamics, haemodynamics, biomechanics or acoustics. Mathematical understanding of coupling conditions in connection with the development, analysis and implementation of substructuring methods becomes more and more important. The aim of this minisymposium is to bring together scientists working in this field to report about recent developments.

Iterative Methods for the Saddle-point Problem Arising from the H_C/E_I Formulation of the Eddy Current Problem

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Abstract: The solution of the linear system arising from a finite element approximation of the time-harmonic eddy current problem is considered. In particular we focus on the formulation that retains as main unknowns the magnetic field in the conductor HC and the electric field in the insulator EI. We propose and analyze iterative procedures based on the physical decomposition of the computational domain in a conducting region and an air region. If the insulator does not contain any non-bounding cycle we prove that the Dirichlet/Neumann iteration converges with a rate that is independent of the mesh size. In the case of a connected conductor with general topology we propose to use either a modified version of the Dirichlet/Neumann iteration or an Uzawa-like method. We compare the performance of both methods by solving four different test problems.

References

FETI for Darcy-Mortar-Stokes Systems
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Abstract: We consider the coupling across an interface of a fluid flow and a porous media flow. The differential equations involve Stokes equations in the fluid region and Darcy equations in the porous region, and coupled through an interface with Beaver-Joseph transmission conditions. The discretization consists of Stokes finite elements in the fluid region, Raviart-Thomas finite elements in the porous region, and mortar Lagrange multipliers on the interface. We allow nonmatching meshes across the interface. The goal of the talk is to discuss some issues of these discretizations and then to discuss optimal and non-optimal preconditioners with respect to mesh parameters, viscosity and permeability. Numerical experiments will be presented to confirm the sharpness of the theoretical estimates.

Heterogeneous Modelling of the Human Knee
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Abstract: A precise prediction of spatially resolved distribution of loads and forces within human joints would support surgical decisions and thus increase the overall success of hip and knee surgery procedures. On the other hand, the numerical simulation of joints requires the mathematical representation of the dynamical interplay of heterogeneous materials, such as bones, muscles, ligaments, tendons, cartilage, and complicated patient-specific geometries in 3D.

In this talk, we will report on recent progress in the simulation of the human knee, and particularly concentrate on efficient and reliable solvers based on heterogeneous domain decomposition and truncated non-smooth Newton multigrid.
Efficient Simulation Techniques for Heterogeneous Models in Mechanics and Biomechanics

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(Joint work with C. Gross and D. Krause)

Abstract: The mathematical modeling of complex (mechanical) models often gives rise to heterogeneous and strongly nonlinear models, whose numerical treatment is far from trivial. In this talk, we discuss different aspects of decomposition approaches for the efficient simulation of heterogeneous and strongly nonlinear models in, e.g., fracture mechanics and biomechanics. On the discretization side, we consider the coupling of as different models as molecular dynamics and continuum mechanics. As a matter of fact, molecular dynamics is formulated in a discrete, Euclidean phase space while for models in continuum mechanics typically infinite dimensional function spaces are used for displacements and stresses. Using techniques from scattered data approximation and non-conforming domain decomposition, we "imbed" the MD phase space in the function space $L^2$, leading to a new weak coupling approach for the scale transfer. We investigate the properties of this coupling approach and discuss the difficulties related to the coupling of the two different models along illustrative numerical examples. On the side of solution methods, we shortly will present a new nonlinearly preconditioned globalization strategy for the parallel solution of strongly nonlinear problems. This strategy allows for combining locally obtained search directions to a global descent direction, which in turn gives rise to a globally convergent solution method. Thus, in case a global objective function can be provided, nonlinear problems can be solved efficiently using nonlinear solution strategies on the respective subdomains. An advantage of our new approach is a reduction of parallel communication and faster convergence in case of localized nonlinearities, which makes it particularly attractive for heterogeneous problems.


**How Close to the Fully Viscous Solution can one get When Inviscid Approximations are Used in Subregions?**

Gander, M. J. 1; Halpern, L. 2; Japhet, C. 3; Véronique, Martin; 4

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**Abstract:** In many applications the viscous terms become only important in parts of the computational domain. As a typical example serves the flow around the wing of an airplane, where close to the wing the viscous terms in the Navier Stokes equations are essential for the solution, while away from the wing, Euler's equations would suffice for the simulation. This leads to the interesting problem of finding coupling conditions between these two partial differential equations of different type. Good coupling conditions should lead to an approximate solution which is as close as possible to the fully viscous one in the entire domain. We are interested in this talk in the one dimensional model problem of advection reaction di®usion equations with pure advection reaction approximation in subregions, which leads to the problem of coupling first and second order operators. We review some previously developed methods ([1], [4], [2]) and present a new approach based on the factorization of the operator, which leads to new sets of coupling conditions for this problem. We compare the different methods both analytically and numerically, and give asymptotic error estimates when the viscosity becomes small. Our results show that the new coupling conditions based on operator factorization lead to approximate solutions which are much closer to the fully viscous solution than the approaches found in the literature.

**References**


**Heterogeneous Domain Decomposition in Dune**

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**Abstract:** DUNE is a set of C++ libraries for grid-based numerical methods. It offers an unseen amount of flexibility without compromising efficiency and usability. The core of DUNE is an abstract interface for grids, which allows to separate grid implementations from the algorithms that use them. Grid implementations can be changed at any moment in the development process without any impact on application code. Hence, for each application the perfect grid implementation can be chosen. Also, with this interface, it becomes straightforward to handle several different grid implementations at the same time. This can be convenient for domain decomposition methods involving subproblems on different grids or on grids of different dimensions.

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Xue, Guangri

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Abstract: Using Kirchhoff transformation, we develop a Dirichlet-Neumann alternating iterative domain decomposition method for a 2D steady-state two-phase model for the cathode of a polymer electrolyte fuel cell (PEFC) which contains a channel and a gas diffusion layer (GDL). This two-phase PEFC model is represented by a nonlinear coupled system which typically includes a modified Navier-Stokes equation with Darcy’s drag as an additional source term of the momentum equation, and a convection-diffusion equation for the water concentration with discontinuous and degenerate diffusivity. For both cases of dry and wet gas channel, we employ Kirchhoff transformation and Dirichlet-Neumann alternating iteration with appropriate interfacial conditions on the GDL/channel interface to treat the jump nonlinearities in the water equation. Numerical experiments demonstrate that fast convergence as well as accurate numerical solutions are obtained simultaneously owing to the implementation of the above-described numerical techniques along with a combined finite element-upwind finite volume discretization to automatically control the dominant convection terms arising in the gas channel.

Multilevel Methods for Complex Fluid Simulations
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Abstract: The link between various viscoelastic fluids models and the symmetric matrix Riccati differential equations can be a new device that brings to unified and proper ways of numerical treatments for the viscoelastic models. In this talk, we describe a few steps toward efficient numerical schemes for complex fluids simulation. First, we construct stable finite element discretizations using Eulerian-Lagrangian methods based on the Riccati formulations of the viscoelastic models. Then we develop a new multilevel time-marching scheme; together with adaptive time-stepping schemes and time parallel schemes, we can build efficient methods for complex fluids simulation. Furthermore, we discuss two robust and efficient multilevel solvers for Stokes-type systems arising at each time step in the Eulerian-Lagrangian discretization.
5. Domain Decomposition Algorithms in Space-Time

Organisers: Halpern, Laurence; Gander, Martin

Abstract: Domain decomposition algorithms have mainly been developed for stationary problems, and the field has reached a certain maturity with Schwarz methods and substructuring methods, like the FETI methods. When solving time dependent problems, there are two classical approaches for parallelization: after a discretization in time with a uniform time step over the entire domain, one applies either a domain decomposition method for the steady problems obtained at each time step by an implicit time discretization, or one simply advances in parallel, if the time discretization is explicit. There are two main drawbacks to this approach: first one needs to use the same uniform time step in the entire domain, and second one needs frequent communication of small quantities of data at each time step, which can be costly on a parallel computer with relatively slow communication channels. Over the last decade, Schwarz waveform relaxation methods were developed as a remedy to both these problems: they solve time dependent problems on subdomains, using local time stepping, and communicate only information to the neighboring subdomains after completion of an entire time window. This minisymposium will show recent advances in this area, including space-time adaptive moving mesh methods, optimized Schwarz waveform relaxation methods for non-linear problems, and large scale applications.

Schwarz Waveform Relaxation Algorithms with Nonlinear Transmission Conditions for Reaction-Diffusion Equations

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Abstract: We are interested in domain decomposition algorithms of Schwarz waveform relaxation type for a nonlinear reaction-diffusion equation. This is a model problem encountered in geological CO2 storage modeling. Schwarz waveform relaxation algorithms are based on a decomposition of the problem in space, like classical Schwarz methods, but they solve subdomain problems in both space and time. This approach is suitable for parallelization and allows us to use different time and space grids in each subdomains.

We define linear and nonlinear Robin and Ventcel transmission conditions between the subdomains, which lead to a well defined algorithm. The nonlinear conditions are based on best approximation problems for the linear equation and provide an efficient algorithm, as we can see by the numerical results that we present.

This research is supported by the research project SHPCO2 funded by ANR-07-CIS7-007-03.
Optimized Schwarz Waveform Relaxation Methods for Advection Reaction Diffusion Problems

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Abstract: Optimized Schwarz waveform relaxation methods are space-time parallel methods based on subdomain decompositions in space-time. For rapid convergence, these methods use transmission conditions adapted to the evolution problem being solved. Such conditions have been developed for a variety of equations, based on two subdomain decompositions, and they lead to convergence on time windows in very few iterations. We present in this talk an extensive numerical study of optimized Schwarz waveform relaxation methods for advection reaction diffusion equations in two dimensions, for arbitrary decompositions including cross points, using new asymptotic formulas for the optimized transmission conditions.

This research is supported by the research project COMMA funded by ANR BLAN06-1_136426.

Recent Advances in Schwarz Waveform Moving Mesh Methods

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Abstract: Realistic (and interesting) mathematical models often involve quantities which vary over disparate time and space scales. Successful simulation necessarily involves a blend of computational techniques and close attention to the details of the interaction between them. In this talk I review three recently proposed algorithms which couple Schwarz Waveform relaxation and moving mesh methods (r-refinement) to solve time-dependent partial different equations. Recent advances in the development of a 2D Schwarz waveform relaxation moving mesh method will also be discussed.
Waveform Domain Decomposition Methods in Space-Time
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Abstract: In this talk, two kinds of Schwarz waveform relaxation methods based on different transmission conditions are considered to solve time-periodic partial differential equations. Firstly, we consider the parabolic problems with time-periodic. For the algorithm with overlapping Dirichlet transmission condition, the convergence is proved and the dependence of contractive rate on overlap is analyzed. We will also show the convergence of the method using the Robin condition on interfaces which allows the parameter in the Robin condition to be dependent on space-time variables. Secondly, the Schwarz waveform relaxation method for the wave propagation equations with time-periodic is introduced. The original problem is decomposed into subproblems coupled with absorbing boundary conditions with overlap and nonoverlap respectively. Then, we solve the periodic subproblems by the controllability method. The proofs of convergence are available. Meanwhile, the discrete Schwarz waveform relaxation method is considered. With the finite difference scheme, we get the convergence too. Numerical experiments, covering convection-dominated, reaction-dominated, variable coefficients, and wave problems, are carried out to show the effectiveness of our methods.

References


An Optimized Schwarz Waveform Relaxation Method for the Linearized Primitive Equations of the Ocean
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Abstract: We are interested in the derivation of efficient domain decomposition methods for the viscous primitive equations of the ocean. We consider the rotating 3d incompressible hydrostatic Navier-Stokes equations with free surface. Performing an asymptotic analysis of the system with respect to the Rossby number, we compute an approximated Dirichlet to Neumann operator and build an optimized Schwarz waveform relaxation algorithm. We establish the well-posedness of this algorithm and present some numerical results to illustrate the method.

This research is supported by the research project COMMA funded by ANR BLAN06-1_136426.

Organizers: Dolean, Victorita; Gander, Martin; Lanteri, Stéphane

Abstract: Electromagnetic devices are ubiquitous in present day technology. Although the principles of electromagnetics are well understood, solving Maxwell's equations is a challenge, both for time-domain and time-harmonic formulations. For practical applications, the solution of such problems is further complicated by the detailed geometrical features of scattering objects, the physical properties of the propagation medium and the characteristics of the radiating sources. In addition, because the wavelength is often short, the algebraic systems resulting from the discretization of Maxwell's equations can be extremely large. Domain decomposition principles are thus ideally suited for the design of efficient and scalable solvers for such systems. This mini-symposium will give an overview of recent developments on domain decomposition solvers for electromagnetic wave propagation problems, with contributions covering both mathematical aspects and practical applications.

Optimized Schwarz Methods for Maxwell's Equations

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Abstract: Over the last few years, optimized Schwarz methods have been developed and analyzed for Maxwell's equations. These methods use combinations of the characteristic variables at interfaces between subdomains in order to obtain faster convergence than the classical methods which use characteristic variables, and the same technique to exclude resonance frequencies as for Helmholtz equations.

In current applications of interest, like the study of the influence of cell phones on the brain, the computational domain is given in part by the human head tissues, where the electromagnetic waves are damped. We develop and analyze optimized transmission conditions for this case, and show that these transmission conditions can be related to the somewhat artificial exclusion of resonance frequencies in the undamped case. We illustrate our findings with numerical experiments.
Research Advances in Domain Decomposition Methods for Electromagnetic Problems

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Abstract: In this talk, the research advances in domain decomposition method (DDM) for complex electromagnetic (EM) problems in the State Key Laboratory of Millimeter Waves of China are reviewed, which include the finite difference (FD) and Laplace equation based DDM for the parameter extraction of VLSI interconnects, the FD and Helmholtz equation based DDM for the two-dimensional (2D) EM scattering problems, the frequency domain FD (FDFD) and Maxwell equation based DDM for three-dimensional (3D) EM problems of multilayered circuit parameter extraction, antenna radiation and object scattering etc., the time domain FD (TDFD) and TD Maxwell equation based DDM for 3D EM radiation and scattering problems, the EM surface integral equation (IE) and method of moments (MoM) based DDM for the EM scattering analysis of electrically large conducting objects, the partial basic solution vector (PBSV) based DDM for 2D and 3D EM guided wave and scattering problems, and the projective decomposition method etc.

A Dual-field Domain Decomposition Method for Time-Domain Finite Element Computation of Electromagnetic Fields

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Abstract: This presentation focuses on a novel domain decomposition algorithm for solving Maxwell’s equations using the time-domain finite element method. In this algorithm, a computation domain is first subdivided into a number of nonoverlapping subdomains, and within each subdomain both electric and magnetic fields are discretized based on their second-order vector wave equations. A leapfrog time-marching scheme is then employed to compute and update the electric and magnetic fields in each time step. The fields in the subdomains are coupled by the equivalent surface currents at the subdomain interfaces. This algorithm requires minimum communications between the subdomains, thus is highly suitable for parallel computations. It is particularly attractive for numerical simulations of finite arrays and large-scale electromagnetic problems. Numerical stability and dispersion error analyses have been conducted and numerical examples will be presented to demonstrate the accuracy and efficiency of the algorithm. A formulation to model dispersive media has also been developed to expand the capability of the algorithm. When this domain decomposition algorithm is applied to the extreme case where every finite element is regarded as a subdomain, it yields an explicit algorithm, which is comparable to the time-domain discontinuous Galerkin method in terms of computational complexity and modeling capability. A preliminary study has been carried out to compare their accuracy and efficiency.
Domain Decomposition Methods with Higher Order Transmission Conditions for Solving Large Electromagnetic Wave Problems

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Abstract: Domain Decomposition Methods (DDMs) have been employed as effective pre-conditioners to further improve upon the vector finite element methods for much larger electrical-size electromagnetic (EM) problems and with faster convergence in iterative matrix solution techniques. However, most of the DDMs use 1st order Robin transmission condition (TC), which does not provide converging mechanism for evanescent modes across domain interfaces. Therefore, the performance of the DDMs tends to deteriorate significantly once the mesh sizes on the interfaces are very small compared to wavelength. This could potentially be a major limitation in treating many real-life engineering problems, since multi-scale nature of the EM devices makes it unavoidable of small mesh sizes. In this paper, we introduce a new full second order TC, which provides converging mechanism for both propagating and evanescent modes. Unlike the recent developed “optimal transmission condition”, the proposed full second order TC does not trade the performance in propagating modes to achieve the convergence in evanescent modes.

A Discrete Weighted Helmholtz Decomposition and its Application to Nonoverlapping DDM for Maxwell Equations

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Abstract: We propose a discrete weighted Helmholtz decomposition in edge element spaces. The decomposition is orthogonal in a weighted $L^2$ inner product and stable uniformly with respect to the jumps in the discontinuous weight function. With the help of such a weighted Helmholtz decomposition, we can show that some existing substructuring preconditioners converge not only nearly optimally in terms of the subdomain diameter and the finite element mesh size, but also independently of the jumps in the coefficients across the interfaces between any two subdomains.
7. The Multipliers-free Dual-primal Domain Decomposition Methods

Organizers: Herrera, Ismael; Yates, Robert A.

The objective of the Minisymposium here proposed is to make an integrated presentation of an approach recently introduced that permits formulating dual-primal domain decomposition methods without recourse to Lagrange multipliers, as it is explained in the Abstract that follows. The Minisymposium consists of six lectures that cover different aspects of these new methods.

Abstract: Some of the most efficient non-overlapping DDMs available at present are the dual-primal versions of the Neumann-Neumann method and of the non-preconditioned FETI [1], here referred to as round trip methods. The treatment of this kind of algorithms, until recently, had been done with recourse to Lagrange multipliers, exclusively. However, Herrera and his collaborators have introduced multipliers-free versions of all these methods, including the one-way algorithms (by this we mean the non-preconditioned FETI and the Schur complement method). The line of research that has led to these developments [2-5] is based on a unified theory linear operators acting on piecewise defined functions, or vectors, that was developed through a long time span (see [2] for references to such previous work). Numerical evidence has been obtained that the new versions perform at least as well as the standard formulations. However, the new formulations are more direct, easier to apply and yield more general explicit matrix-expressions.

References

Comparisons with Standard Formulations

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Abstract: The FETI Roundtrip (RT) method and the Finite Element Tearing and Interconnecting (FETI) method are compared in this talk. FETI is based upon a variational approach and utilizes Lagrange multipliers to enforce continuity on the internal boundary while RT is based directly (without resource to Lagrange Multipliers) upon a sequence of transformations between subspaces of the discontinuous function space. While both methods yield similar numbers of iterations for convergence, RT is derived directly from the original operator and does not require Lagrange multipliers. RT works with no changes on both symmetric and non-symmetric operators. Various test problems are presented to illustrate the similarities and differences between FETI and RT solvers. The following points are highlighted:

1. RT is derived directly from the original operator and does not use Lagrange multipliers
2. The j operator used in the RT method appears to be the “optimal” B operator used in FETI
3. RT is easier to program than FETI (fewer operators, projections are required)
4. RT works intact on non-symmetric problems
5. RT can work easily with standard, independent subdomain solvers, requiring only $\mathcal{A}^{-1}$ computations in each subdomain.

References

**Multipliers-Free Parallel Algorithms for Elastic Systems**

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**Abstract:** Mathematical models of many systems of interest, including very important continuous systems of Engineering and Science, lead to a great variety of partial differential equations whose solution methods are based on the computational processing of large-scale algebraic systems. A conspicuous example are elastic systems models, for which usually the number of degrees of freedom involved is especially great, since they are not governed by a single partial differential equation but instead by systems of such equations. Fortunately, the barrier posed by this awesomely large number of degrees of freedom has been overcome in the last decades by the incredible expansion of human computational capacity, which has made amenable to effective treatment an ever increasing diversity and complexity of problems occurring in engineering and scientific applications. Parallel computing is outstanding among the new computational tools, especially at present when further increases in hardware speed apparently have reached insurmountable barriers, and so the emergence of parallel computing during the last twenty years or so, prompted on the part of the computational-modeling community a continued and systematic effort with the purpose of harnessing it for the endeavor of solving partial differential equations. Very early after such an effort began, it was recognized that domain decomposition methods (DDM) are the most effective techniques for applying parallel computing to the solution of partial differential equations, since such an approach drastically simplifies the coordination of the many processors that carry out the different tasks and also reduces very much the requirements of information-transportation between them. There are many approaches to DDM, albeit lately much of the effort has gone into iterative substructuring methods in non-overlapping partitions, mainly because they are more effective for many problems [1]. A direct application of such an approach yields the Schur-complement method and the non-preconditioned FETI method (referred to generically as one-level or ‘single-trip methods’ [2-5]). The performance of these methods, however, usually is not satisfactory and can be drastically improved by applying appropriate preconditioners [1]. Some of the most efficient preconditioned non-overlapping methods are obtained by using the Neumann method as a preconditioner of the Schur-complement method; or, conversely, using the Schur-complement method as a preconditioner of the Neumann method. This gives rise to the ‘round-trip (or, two-level) methods’ [5], which include the Neumann-Neumann and the preconditioned FETI methods, whose performance is further improved by incorporating dual-primal preconditioners [6-9]. In the processing of such methods, discontinuous functions are introduced at some stages. Until recently, the treatment of discontinuous functions had been based on the use of Lagrange multipliers (see [1], for a review of this topic). However, an approach to dual-primal domain decomposition methods that is formulated without recourse to Lagrange-multipliers, has just been introduced by Herrera and his co-workers [2-5]. In it, the algorithms are derived directly from the problem-matrices, independently of the partial differential equations that originated them and the number of dimensions of the problem. It, furthermore, yields robust and easy-to-construct computer codes that reduce considerably the code-development effort required for their implementation; the new method permits, for example, transforming 2-D codes into 3-D codes easily. The systematic use of the average and jump matrices yields significant advantages due to their superior algebraic and computational properties, which can be effectively applied not only at internal-boundary-nodes, but also at edges and corners. One-level methods and two-level methods are incorporated in a unified manner by the new approach. For all these methods explicit matrix expressions are supplied [5], which by simple substitutions yield parallel-processing codes for any system that is governed by linear differential equations, or by systems of such equations, that are symmetric and non-negative. Thus far, most of the applications of the new methodology have referred to systems governed by a single differential equation. Here, we apply it to the system of equations of linear elasticity. In research now underway, its performance is being compared with other DDM elastic-systems models.

**References**

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DDM Applied To Contaminant Transport

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Abstract: Recently various mathematical and numerical models were developed at our group, applying single-degree of freedom collocation localized method, and finite element methods, for the Transport in porous media. The main objective at this work is to implement a domain decomposition method for those methods applied to the transport equation. On the base of this, it will be possible to apply it to the NAPL transport equation.
The Multipliers-free Dual-Primal DDMs: An Overview

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Abstract: An approach to dual-primal domain decomposition methods is presented, which yields robust and easy-to-construct computer codes that reduce considerably the code-development effort required for their implementation [1-4]. The new formulations permit, for example, transforming 2-D codes into 3-D codes easily. The systematic use of the average and jump matrices yields significant advantages due to their superior algebraic and computational properties. These matrices can be effectively applied not only at internal-boundary-nodes but also at edges and corners; they constitute generalizations of the ‘average’ and ‘jump’ of a function and are themselves a contribution of the new approach to the methods of domain decomposition. One-level methods such as Schur-complement and one-level FETI, and two-level methods such as Neumann-Neumann and preconditioned FETI, are incorporated in a unified manner by the new approach, which contrary to standard approaches is formulated without recourse to Lagrange-multipliers. For all these methods explicit matrix expressions are given in Article 199, which concludes and summarizes such developments. In another paper, now going to press, these procedures are extended to non-symmetric matrices.

References

Multipliers-Free General Theory of Partial Differential Operators Acting on Discontinuous Functions and of Matrices Acting on Discontinuous Vectors

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Abstract: Truly general and systematic theories of Finite Element Methods (FEM) and of Domain Decomposition Methods (DDM) should be formulated using, as trial and test functions, piecewise-defined-functions that can be fully discontinuous across the internal boundary which separates the finite elements from each other. When this is done, such a FEM theory includes discontinuous Galerkin (dG) method as a particular case. As for domain decomposition methods, the non-overlapping substructuring approaches stem from combinations of two basic one-level procedures: the Schur-complement and the FETI methods. In the first one of these methods one poses Dirichlet problems with continuous data in the internal boundary of the domain-partition, while Neumann problems with continuous normal derivatives are posed in the second one. Then, such problem-solutions corresponding to the first one of these methods have discontinuous normal derivatives; on the other hand, in the second one of these methods the functions that are discontinuous are the solutions themselves. Thus, DDMs also require treating fully-discontinuous functions. However, both in FEM and DDM the approach to discontinuous functions is based on the introduction of Lagrange-Multipliers [1,2,3]; i.e., discontinuous functions are treated as an anomaly that has to be cured by the use of Lagrange-Multipliers. However, a multipliers-free approach to discontinuous functions, without recourse to Lagrange-multipliers, has just been introduced by Herrera and his co-workers [4,5,6,7,8] that has been applied in both, FEM and DDM formulations. This talk, as reflected in its title, is devoted to present ‘The Multipliers-Free General Theory of Partial Differential Operators Acting on Discontinuous Functions and of Matrices Acting on Discontinuous Vectors’ that unifies both formulations.

References

The Multiplier-Free Dual-Primal Domain Decomposition Methods: Implementation Issues

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Abstract: The multipliers-free dual-primal DDMs offer a new set of algorithms for the numerical solution to PDEs [1-4]. These same algorithms, in fact, with no significant modifications, can handle both symmetric and non-symmetric operators [5]. Certain care, however, must be taken during the implementation process to insure, first of all, that the matrices are properly constructed in the subdomains and that the different operations involved are executed in an efficient manner. A second issue is to ensure that a high degree of parallelism be obtained. Since the algorithms are applied directly to a given linear system of equations without recourse to Lagrange multipliers, the programming logic can be organized to utilize different solution techniques such as the Conjugate Gradient Method for symmetric systems and GMRES for non-symmetric systems. Furthermore, predefined subdomain solvers can be incorporated without much difficulty. Finally, little, if any, changes are required to process problems in different space dimensions. Consequently, it is possible to develop relatively simple, yet highly robust computer codes with the use of these methods.

References

8. Hybrid Domain Decomposition Methods for Multiphysics

Organizers: Langer, Ulrich; Steinbach, Olaf

Abstract: The coupling of different physical models leads in a natural way to the application of domain decomposition methods for the discretization and for efficient solution algorithms. Within this minisymposium the focus will be on the use of hybrid domain decomposition methods to couple different local discretization strategies such as finite and boundary element methods, and on the use of tearing and interconnecting solution strategies.

Domain Decomposition Solvers for Nonlinear Multi-Harmonic Finite Element Equations

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Abstract: In many practical applications, for instance, in computational electromagnetics, the excitation is time-harmonic. Switching from the time domain to the frequency domain allows us to replace the expensive time-integration procedure by the solution of a simple elliptic equation for the amplitude. This is true for linear problems, but not for nonlinear problems. However, due to the periodicity of the solution, we can expand the solution in a Fourier series. Truncating this Fourier series and approximating the Fourier coefficients by finite elements, we arrive at a large-scale coupled nonlinear system for determining the finite element approximation to the Fourier coefficients. The construction of fast solvers for such systems is very crucial for the efficiency of this multi-harmonic approach. In this talk, we look at non-linear, time-harmonic potential problems. We construct and analyze almost optimal solvers for the Jacobi systems arising from the Newton linearization of the large-scale coupled nonlinear system.
Scalable FETI/BETI Algorithms for 3D Contact Problems of Elasticity

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(Joint with T. Brzobohaty, T. Kozubek, A. Markopoulos, M. Sadowska and V. Vondrak)

Abstract: We first briefly review the TFETI/TBETI (total finite/boundary element tearing and interconnecting) based domain decomposition methodology adapted to the solution of 2D and 3D frictionless multibody contact problems of elasticity. Recall that unlike the original FETI/BETI method, which assumes that the subdomains inherit the prescribed Dirichlet boundary conditions from the definition of the problem, TFETI/TBETI imposes the prescribed displacements by the Lagrange multipliers, so that all the subdomains are floating and their kernels are a priori known. Then we show that the “natural coarse grid” of the rigid body motions defines a projector to the subspace of Lagrange multipliers with the solution. Moreover, the preconditioning by the projector reduces the condition number of the dual Schur complement so that it is independent on the discretization parameter h and accelerates also the non-linear steps. The procedure results in the quadratic programming problem in the dual variables with a well conditioned Hessian matrix, homogeneous equality constraints, and bound constraints.

Then we review our in a sense optimal algorithms for the solution of the resulting quadratic programming problems. The algorithms fully exploit the specific structure of these problems. The unique feature of these algorithms is their capability to solve the class of convex quadratic programming problems with homogeneous equality constraints and bound constraints in \( O(1) \) iterations provided the spectrum of the Hessian of the cost function is in a given positive interval. The theory yields the error bounds that are independent of conditioning of the constraints.

Finally we put together the above results to develop scalable algorithms for the solution of both coercive and semi-coercive variational inequalities. The theory covers also the problems with “floating” bodies. Rather surprisingly, the results are qualitatively the same as the classical results on scalability of FETI for linear elliptic problems. We give results of numerical experiments with parallel solution of both coercive and semicoercive 2D and 3D contact problems discretized by up to more than 10 million of nodal variables to demonstrate that the scalability can be observed in computational practice. The power of the results is demonstrated also by the solution of difficult real-life problems as analysis of ball bearings.
**Coupled FE/BE Formulations for the FSI**

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**Abstract:** The coupling of finite and boundary element methods is well suited to handle different physical models and phenomena, in particular when including exterior boundary value problems. Here we will focus on the time-harmonic acousto-structure interaction where the acoustic field is modeled by using boundary integral equations which are stable for all wave numbers.

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**Robust BE Domain Decomposition Methods in Acoustics**

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**Abstract:** In this talk we will present a boundary element tearing and interconnecting approach for the Helmholtz equation. In contrary to the Laplace equation it is in general not known if local Dirichlet or Neumann problems admit a unique solution. So one has to stabilize the standard approach to get rid of artificial eigenfrequencies of the local problems. In this talk we will present a stabilized approach which leads to a uniquely solvable discrete system. This will be done in two steps: First Robin boundary conditions are introduced to ensure the solvability of the local problem. But the Steklov-Poincare operator, which is used in the formulation may not well defined if the local Dirichlet problem is not uniquely solvable. So we introduce an alternative formulation for the local problem which leads to an always well defined and uniquely solvable formulation. Additionally, one can prove that also the discrete local and the discrete global problem have a unique solution.

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**Numerical Simulations of Fluid-Structure-Interaction (FSI) Problems on Hybrid Meshes with Algebraic Multigrid Methods**

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**Abstract:** Fluid-structure interaction problems arise in many application fields such as flows around elastic structures or blood flow problems in arteries. The method presented in this paper for solving such a problem is based on a reduction to an equation at the interface, involving the so-called Steklov-Poincaré operators. This interface equation is solved by a Newton-like iteration for which directional. One step of the Newton-like iteration requires the solution of several decoupled linear subproblems in the structural and the fluid domains. These subproblems are spatially discretized by a finite element method on hybrid meshes. For the time discretization implicit first-order methods are used for both subproblems. The discretized equations are solved by algebraic multigrid methods.
9. Theory and Application of Adaptive and Multilevel Methods

Organizers: Xu, Jinchao; Holst, Michael; Hu, Jun; Chen, Long

Abstract: Rational and Plan. Multigrid and adaptive methods are two distinct classes of modern numerical methods for solving partial differential equations. The theories for both types of methods have been studied, essentially independently, for several decades. The analysis of these two type of methods have been done separately, but striking similarities in both the methodology and the analysis have begun to emerge. The mini-symposium is to bring together experts in adaptive finite element method or multigrid methods to discuss new types of questions at the foundation, overlap and application of these two research fields.

A Multilevel Preconditioner for the Crouzeix-Raviart Finite Element Method for Elliptic Problems with Discontinuous Coefficients

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Abstract: In this paper, we propose a multilevel preconditioner for the Crouzeix-Raviart finite element approximation of second order elliptic partial differential equations with discontinuous coefficients. Since the finite element spaces are nonnested, weighted intergrid transfer operators, which are stable under the weighted $L^2$ norm, are introduced to exchange messages between different meshes. By analyzing the eigenvalue distribution of the preconditioned system, we prove that except a few small eigenvalues, all the other eigenvalues are bounded below and above nearly uniformly with respect to the jump and mesh size. As a result, we get that the convergence rate of the preconditioned conjugate gradient method is quasi-uniform with respect to the jump and mesh size. Numerical experiments are presented to confirm our theoretical analysis.

Optimal Convergent Rate in $L^2$-Norm of An Adaptive Finite Element Method for Elliptic Equations

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Abstract: This talk is devoted to an adaptive finite element algorithm is developed for second order elliptic equations to control the error in $L^2$-norm. It complements the standard adaptive finite element method with a procedure to control the mesh size according to the a priori information on the second derivative of the solution. Optimal convergent rate of the error in $L^2$-norm for convex domains in both two and three domains and for polygonal domains in two dimensions is ensured by our algorithm.
**Inexact Solvers for Saddle-point System Arising from Domain Decomposition of Linear Elasticity Problems in Three Dimensions**

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**Abstract:** In this paper, a domain decomposition method with Lagrange multipliers based on geometrically non-conforming subdomain partitions for three dimensional linear elasticity is considered. Because of the geometrically non-conforming partitions, appropriate multiplier spaces should be chosen, and a saddle-point system is then built. An augmented technique is introduced, such that the resulting new saddle-point system can be solved by the existing iterative methods. Two simple inexact preconditioners are constructed for the saddle-point system, one for the displacement variable, and the other for the Schur complement associated with the multiplier variable. It is shown that the global preconditioned system has a nearly optimal condition number, which is independent of the large variations of the material parameters across the local interfaces.

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**Convergence and Optimality of Adaptive Nonconforming Methods for High-Order Partial Differential Equations**

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**Abstract:** This talk is devoted to the convergence and optimality analysis of adaptive finite element methods for a class of nonconforming elements for both the second and the fourth order elliptic problems. Because of the lack of Galerkin-orthogonality, a quasiorthogonality is discovered by using a very special conservative property of this class of nonconforming methods. By introducing a new parameter-dependent error estimator and further establishing a discrete reliability property, sharp convergence and optimality estimates are then fully proved within one framework for both the second order and the fourth order problems in both two and three dimensions. This study improves the existing results for the second order problem and provides the first analysis for the fourth order problems.
Local and Parallel Algorithms for Fourth Order Problems Discretized by the Morley-Wang-Xu Finite Element Method

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Abstract: This talk is focused on Local and parallel algorithms for fourth order problems discretized by the Morley-Wang-Xu finite element method. Due to nonnestedness nature of this element, certain intergrid transfer operators have to be set up to achieve an improved local solution from the global coarse grid solution and the local fine grid correction.

Two types of parallel algorithms have been developed. The first algorithm is similar to Algorithm A1 in [J. Xu and A. Zhou, Local and parallel finite element algorithms based on two-grid discretizations, Math. Comp., 6(1999), 881-909], but the global solution does not belong to the global nonconforming finite element space. To overcome this difficulty, the second one is proposed based on the local algorithm and the partition of unity method. Under certain conditions for solution regions, it is proved that both methods enjoy the discrete energy error of the size \( \alpha_h + H_\infty((h^{-1/2})^d \), where \( H, h, \) and \( d \) denote the mesh sizes of the coarse and fine finite element triangulations, and the space dimension, respectively. Numerical results are included to support the theory obtained.

Parallel Uzawa Algorithm for Maxwell Saddle-point Systems of Higher Order Edge Element

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Abstract: In this paper, we design a preconditioned parallel Uzawa algorithm (HX-ho-Uzawa-p) for solving the saddle-point system generated by higher order edge element discretization of Maxwell equations. In the Uzawa algorithm, we use HX preconditioner for the primal variable. The numerical results indicate that the new algorithm has good parallelization scalability and the iteration counts is independent of mesh size and jump range no matter including floating subdomains.
Uniform Convergence of Multigrid V-cycle on Adaptively Refined Finite Element Meshes for Elliptic Problems with Discontinuous Coefficients

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Abstract: The uniform convergence of the standard multigrid V-cycle algorithm with Gauss-Seidel relaxation performed only on new nodes and their “immediate” neighbors for adaptive finite element discretizations of elliptic problems with “quasi-monotone” discontinuous coefficients on local refined meshes using the newest vertex bisection algorithm is proved. The multigrid V-cycle algorithm uses O(N) operations per iteration and is optimal.

Optimal Multilevel and Adaptive Finite Element Methods for Time-harmonic Maxwell Equations

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Abstract: Three related results in this paper will be presented for the finite element discretization of time-harmonic Maxwell equations in three dimensions with moderate size of frequency: an optimal L2 estimate, a class of two-grid methods and convergence and complexity of adaptive edge finite element methods. Numerical experiments are carried out to justify the optimality of these results.
Multi-level Boolean Sum Corrections in Finite Element Approximations

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Abstract: Based on the Boolean sum technique, in this presentation, we introduce a class of multi-level iterative corrections for finite element approximations. This type of multilevel corrections can produce highly accurate approximations. For illustration, we present some old and new finite element correction schemes for an elliptic boundary value problem.

A Posteriori Hierarchical Analysis for an Obstacle Problem

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Abstract: A posteriori analysis for the energy norm of the finite elements approximate error of elliptic obstacle problems is established in the framework of hierarchical approach. It is shown that up to some high order oscillation terms of the data, the energy norm of the local errors within the discrete incipient area are equivalent to the classical edge-oriented hierarchical error estimators designed for unconstrained linear equations; the contributions of the local error in the so-called full-contact discrete coincident area are 0 and the local errors within the other discrete coincident area are equivalent to some positive edge-oriented hierarchical estimators plus some element-oriented hierarchical estimators.
Contributed Talks

Probabilistic Domain Decomposition of Nonlinear Parabolic
Partial Differential Equations by Random Trees

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Abstract: A domain decomposition method is developed for solving numerically nonlinear parabolic partial differential equations, based on the probabilistic representation of solutions. Such a direct probabilistic representation requires generating a number of random trees, whose role is that of the realizations of stochastic processes used in the linear problems. First, only few values of the sought solution inside the space-time domain are computed (by a Monte Carlo method on the trees), and an interpolation is then accomplished, in order to approximate interfacial values of the solution inside the domain. Finally, a fully decoupled set of sub-problems is obtained. The algorithm is suited to massively parallel implementation, enjoying arbitrary scalability and fault tolerance properties.

Subspace Correction Methods for Interior Penalty Discontinuous
Galerkin Approximations of Elliptic Problems.

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Abstract: In this talk, we present some new iterative and preconditioning techniques for the solution of the linear systems resulting from discontinuous Galerkin (DG) Interior Penalty (IP) discretizations of elliptic problems. The proposed iterative methods are designed using the subspace correction framework and are based on a “natural” decomposition of the first order DG finite element space as a direct sum of the Crouzeix-Raviart non-conforming finite element space and a subspace that contains functions discontinuous at interior faces. We show uniform convergence of the proposed algorithms for both symmetric and non-symmetric IP schemes. We also present some numerical experiments to validate the theory and to assess the robustness of the proposed methods.
A Multigrid Continuation Algorithm for Multiple Peak Solutions of the Gross-Pitaevskii Equation in a Periodic Potential

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Abstract: We describe a staircase multigrid-continuation algorithm (SMCA) for computing energy levels of Bose-Einstein condensates (BEC) in a periodic potential. The proposed algorithm is a modification of the two-grid discretization schemes or the simplified two-grid schemes which has the following advantages over the latter: (i) It guarantees that the scheme will converge to the target point on the finest grid. (ii) It is cheaper than the simplified two-grid scheme. We apply the proposed algorithm to compute the ground state and the first-few excited-state solutions of the 1D, 2D and 3D BEC in a periodic potential. In particular, we compare the performance of the SMCA with the imaginary time evolution method (ITEM), ITEM with steepest gradient (ITS), accelerated ITEM (AIITEM), and AIITEM with amplitude normalization (AIITEM (A.N.)). Our numerical results show that probably the SMCA is not as efficient as the ITS, at least it takes less Newton iterations to reach the target point.

Parareal Algorithm For Hamiltonian Systems

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Abstract: The parareal algorithm allows to use efficiently parallel computers for the simulation of time dependant problems. It is based on a decomposition of the time propagation interval into subintervals, and the propagation over each subinterval is done concurrently on the different processors.

In this presentation, we shall present a generalization of this algorithm that allows to cure a difficulty of the plain version when applied to the simulation over a very long time of Hamiltonian systems. Indeed for these systems, the preservation of some invariant quantities is crucial for these long simulations and the plain parareal algorithm does not maintain well them leading to divergence of the trajectories.

We shall explain the modifications that are quite easy to implement and illustrate the good behavior of this new scheme over a series of numerical examples. This is a joint work with C. Lebris, F. Legoll and Y. Maday.
Schwarz Waveform Relaxation Methods for Systems of Semi-Linear Reaction-Diffusion Equations
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Abstract: Schwarz waveform relaxation methods have been extensively studied for scalar linear partial differential equations (PDEs) of parabolic and hyperbolic type. These algorithms are based on a space-time decomposition and an iteration, where subdomain problems are solved in space-time on overlapping subdomains at each iteration.

For non-linear problems, there are only few convergence studies, for a scalar semi-linear reaction diffusion equation, and for Burgers equation. We present here a first convergence analysis for semi-linear systems of reaction diffusion equations. We prove that the algorithm converges superlinearly on finite time intervals, and also has a linear bound on the convergence rate under an additional hypothesis. We illustrate our analysis with numerical experiments.

On Multilevel Preconditioners Based on Non-nested Meshes
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Abstract: In this talk, we are concerned with the development of multilevel pre-conditioners for linear problems. As opposed to, e.g., standard multigrid methods, the specific objective of this study is to deal with a multilevelhierarchy of unrelated, especially non-nested finite element spaces. In this case, the choice of operators for prolongation and restriction is by no means straightforward.

For the purpose of developing multilevel methods based on non-nested meshes, we present a pseudo-$\mathcal{L}_2$-projection allowing for an efficient information transfer between non-nested meshes. Stability and approximation properties of the new operator are analyzed. Employing these findings within a rather sophisticated construction of an appropriate space hierarchy, we obtain convergence results for both a multiplicative and an additive algorithm. In addition, we review diverse possible prolongation operators and their uses in the given context.

Concluding the presentation, we will show several numerical results of three dimensional applications.
A New Domain Decomposition Algorithm in Coupling Finite Element and Boundary Element Methods

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Abstract: Much benefit can be gained by coupling the finite element (FE) and boundary element (BE) methods. The BE method is more suitable in cases of large or infinite domains because only the boundary of the domain has to be discretized. In combining FE and BE methods the iterative procedure of domain decomposition method can be utilized. The governing equations are analyzed separately for each sub-domains and the equilibrium and compatibility conditions at the interface of sub-domains are satisfied. In this paper, a new method called “Simultaneous Schwarz Neumann-Dirichlet Scheme” has been employed which is based on two relaxation parameters. As is known, a sub-domain cannot be analyzed when the boundary condition applied to it is all of Neuman type. The suggested method removes this limitation. The procedures used in two existing algorithms have also been corrected. Examples have been provided to show the efficiency and advantages of the suggested algorithms over the others.

Multiscale Finite Element Methods for High-contrast Problems

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Abstract: In this poster, we present multiscale finite element methods and domain decomposition methods for multiscale problems in high-contrast media. Both methods use coarse spaces to achieve efficiency and robustness. We investigate coarse spaces designed for high-contrast problems. In particular, basis functions are constructed using solutions of a local spectral problem. In our previous work, we show that using these coarse spaces one can construct preconditioners such that the condition number of the preconditioned system is independent of the contrast. In this poster, these coarse spaces are used to solve elliptic equations with high-contrast heterogeneous coefficients on a coarse grid. Our numerical results show that multiscale finite element methods with coarse spaces constructed via local spectral problems are more accurate compared to multiscale methods that employ "traditional" multiscale spaces.
A Least-Squares/Fictitious Domain Method for Linear Elliptic Problems with Neumann or Robin Boundary Conditions

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(Joint with Glowinski, Roland and Wang, Xiao-Ping)

Abstract: A least-squares/fictitious domain method for the solution of linear elliptic boundary value problems with Neumann or Robin boundary conditions will be discussed. Let \( \Omega \) and \( \Omega' \) be two bounded domains of \( \mathbb{R}^d \) such that \( \Omega' \subset \Omega \). For a linear elliptic problem in \( \Omega', \sigma \) with Neumann or Robin boundary conditions, our goal is to develop a fictitious domain method where one solves a variant of the original problem on the full \( \Omega \), followed by a well-chosen correction on \( \sigma \). This method is of the virtual control type and relies on a least-squares formulation making the problem solvable by a conjugate gradient algorithm operating in a well chosen control space. Numerical results of this method for two dimensional elliptic problems and extension to time dependent problems are given.

Numerical Simulation of Three-dimensional Blood Flows in Arteries Using Domain Decomposition Based Scientific Software Packages in Parallel Computers

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Abstract: A good simulation tool based on patient-specific anatomy and physiologic conditions can be clinically used to help physicians or researchers to study vascular diseases, to enhance diagnoses, as well as to plan surgery procedures. In this paper, we focus on developing parallel domain decomposition algorithms for solving nonlinear systems arising from the discretization of blood flow model equations, where a stabilized finite element method is used for the spatial discretization, while a multistep ODE integrator for the temporal discretization. In particular, at each time step, the resulting system solved by the Newton-Krylov-Schwarz algorithm. We implement the parallel fluid solver using PETSc and integrate it with other software packages into a parallel blood flow simulation system, including Cubit, ParMETIS and ParaView for mesh generation, mesh partitioning, and visualization, respectively. We validated our parallel code and investigated the parallel performance of our algorithms for both a straight artery model and an end-to-side graft model.
A FETI-DP Algorithm for the Three Dimensional Stokes problem
Without Primal Pressure Unknowns

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Abstract: A FETI-DP algorithm for the three dimensional Stokes problem without primal pressure unknowns is developed and analyzed. It is an extension of the author’s previous work on the two dimensional Stokes problem. By solving local Stokes problems, all the pressure unknowns can be eliminated. Velocity averages on common faces are selected as the primal unknowns for the three dimensional problem. This results in a symmetric and positive definite coarse problem. A condition number bound, $O(H/h)$, is proved for the FETI-DP algorithm with a lumped preconditioner, where $H/h$ is the number of elements across a subdomain.

An Error Estimate of the DtN Finite Element Method for
Multiple Scattering Problems

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Abstract: The Dirichlet-to-Neumann (DtN) boundary condition was introduced by Grote-Kirsch [J. Comput. Phys. 201 (2004)] for multiple scattering problems of the acoustic wave, and is called the multiple DtN boundary condition. This boundary condition is analytically represented by an infinite Fourier series.

We present an a priori error estimate of the finite element method applied to the Helmholtz problem with the multiple DtN boundary condition. Our error estimate includes the effect of truncation of the multiple DtN boundary condition as well as that of discretization of the finite element method.
Is the Additive Schwarz method with Harmonic Extension just Parallel Schwarz in disguise?
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Abstract: The Additive Schwarz Method with Harmonic Extension (ASH) was introduced by Cai and Sarkis (1999) as an efficient variant of the additive Schwarz method that converges faster and requires less communication. In this talk, we will show how ASH, which is defined at the matrix level, can be reformulated as an iteration that bears a close resemblance to the parallel Schwarz method of Lions at the continuous level, provided that the decomposition of subdomains contains no cross points. In fact, the iterates of ASH are identical to the iterates of the discretized parallel Schwarz method outside the overlap, whereas inside the overlap it is a linear combination of previous Schwarz iterates. Thus, when used as iterative methods rather than preconditioners, the two methods converge with the same asymptotic rate, unlike the additive Schwarz method, which fails to converge inside the overlap.

Optimized Schwarz Methods to Solve Bi-harmonic Problem
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Abstract: Solving bi-harmonic problem requires elaborate and efficient domain decomposition methods. In this talk we will look at the optimized transmission conditions to construct robust algorithms for solving bi-harmonic differential equations on the plane. First, we will discuss the classical Schwarz method for the bi-harmonic problem and shows how the convergence is slow, especially when the overlap between sub-domains is very small. Secondly, we will investigate different type of transmission conditions between two sub-domains to accelerate the rate of convergence and thus obtained optimized domain decomposition methods which are much faster than the classical Schwarz methods. Some preliminary numerical results are provided.
On Transformation Methods and Their Induced Parallel Properties in Temporal Domain Computation

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Abstract: Many engineering and applied science problems are described by time dependent nonlinear partial differential equations. Numerical methods of handling transient problems are usually based on temporal integration methods such as Euler’s method, Runge-Kutta methods, multi-step methods, etc. In relation to the nature of a given problem which may or may not require fine solution details at intermediate time steps, one usually has to choose a fine or a coarse time stepping. In the case of fine details are required the traditional method is to use temporal integration methods with fine time steps. These temporal integration methods are very difficult to parallelise because of their intrinsic sequential properties. In the case where fine details are not required it is still not possible to use a very large time step in an implicit scheme. There are restrictions imposed on the temporal step size usually due to stability criteria of an explicit scheme or the truncation errors of an implicit scheme in approximating the temporal derivatives. Computing time of such numerical methods inevitably becomes significant. There are also many problems which require solution details not at each time step of the time-marching scheme, but only at a few crucial steps and the steady state. Therefore effort in finding fine details of the solutions using many intermediate time steps is considered being wasted. Such effort becomes significant in the case of nonlinear problems where a linearisation process, which amounts to an inner iterative loop within the time-marching scheme, is required. It would be a significant save in computing time when the linearisation process and the time-marching scheme can both be done in parallel. The main objective of the present work is to remove the time stepping and to combine it with parallel/distributed computers.

To investigate the parallelisation of the temporal domain, this talk begins with a concise overview of classical temporal integration methods, including time-stepping restrict-ions of an explicit scheme, truncation errors in an implicit scheme, and other advantages and disadvantages of using a time marching scheme, and a brief discussion is given of several attempts by various researchers in parallelising temporal integration methods. Second the use of transformation methods and their relations to possibly induce parallel properties to certain intrinsic sequential problems are examined. These transformation methods include the Boltzmann transformations, general stretch transformations, Fourier transformation, and Laplace transformation. Several examples related to these transformations are discussed, including diffusion-convection and image processing problems. Finally, discussions and conclusions are presented.
A Domain Decomposition Method based on Augmented Lagrangian with a Penalty Term in Three Dimensions
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Abstract: In our earlier work [Numer. Math. 112 (2009), 89–113], a dual iterative substructuring method for two dimensional problems was proposed, which is a variant of the FETI-DP method based on the way to deal with the continuity constraint on the interface. The FETI-DP method introduces Lagrange multipliers to enforce the pointwise matching condition on the interface. On the other hand, the proposed method imposes the continuity by not only the pointwise matching condition but also using a penalty term which measures the jump across the interface. For a large penalization parameter, it was proven that the condition number of the resultant dual problem is bounded by a constant independent of both the subdomain size H and the mesh size h. In this presentation, we introduce an extension of a dual substructuring method with a penalty term to three dimensional problems. For the extension to three dimensional case, we mainly consider two things; one is to construct a strong penalty term in 3D enough to guarantee the same convergence speed as in 2D and the other is how to treat an ill-conditioned property of subdomain problems due to the adoption of a large penalization parameter in a penalty term. In both of two key issues, emphasis is placed on the awareness of difference between 2D and 3D in the geometric complexity of an interface. Since edges in 3D make all nodes on the interface coupled unlike in 2D, both of the penalty term and the preconditioner for subdomain problems used in 2D are less efficient in practical sense. Based on such observations, we suggest the modified penalty term and preconditioner in a manner of reducing couplings between functions on the interface. Finally, numerical results are presented.

Leap-frog Mixed Finite Element Methods for Maxwell’s Equations in Dispersive Media
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Abstract: In this talk, we show a systematical way for developing leap-frog mixed finite element methods for solving Maxwell’s equations in dispersive media such as cold plasma, Debye, and Lorentz models. Our schemes are similar to the popular Yee’s FDTD scheme used in electrical engineering community, and is preferable for 3D large scale modeling since no storage of the large coefficient matrix is needed. Conditional stability and optimal error estimate for the proposed schemes are proved. Numerical results justifying the analysis are presented.
The Domain Decomposition Methods Combining Boundary Element with Meshless Local
Petrov-Galerkin Method
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Abstract: Two non-overlapping domain decomposition iterative algorithms combining the boundary
element with meshless local Petrov-Galerkin method are presented in this paper. For speeding up the
convergence rate, static and dynamic relaxation parameters are employed, and the convergence ranges with
the static relaxation parameters are studied. The validity of the dynamic relaxation parameters for the both
algorithms is verified by several numerical examples.

Time DDM for ODEs with Aitken-Schwarz and Time-reversible Schemes
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Abstract: We propose a time domain decomposition method based on the Schwarz algorithm that breaks
the sequentiality of the integration scheme for system of ODEs. For system of linear ODEs (Helmholtz
equations for example), the algorithm shows a linear divergence or convergence allowing to apply the
Aitken's acceleration of the convergence technique to obtain the solution at the boundaries of the
subdomains. A second approach is then developed using time-reversible integration scheme and a system of
conditions satisfied by the subdomains's solution. In case of non linear system of ODEs, the system is solved
by a Newton method. We discuss some problems that arise solving the system of constraints.
Implementation, numerical results and efficiency will be discussed and illustrated by examples.
A Dynamic Error Control Scheme for Numerical Solution of Differential Equations

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Abstract: With a good algorithm in hand, a design engineer or a practitioner often faces a challenging question when trying to solve a differential equation: what step size should I use so that the solution can be efficiently computed and that the solution will satisfy the quality control criteria? We derive a simple yet dynamic error control formula based purely on the computed solutions. The idea is from the Richardson extrapolation using asymptotic expansion. A good practical feature of the scheme is that it is almost independent of the method to be used and the differential equation to be solved. One numerical example with known solution is included to demonstrate the effectiveness of this scheme, and another numerical example from circuit design is presented to show how to use it in practice.

A Parallel Direct Solver for Large Scale and Sparse Linear System of Equations

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Abstract: A fast parallel direct solver for large scale sparse linear system of equations is presented in this paper. It can be regarded as a combination of Domain Decomposition method and multigrid method; As a direct solver, this method is among the most efficient direct solvers available so far with flops count as O(log n). This method possess a huge advantages over the existing fast solver in which it can be used to handle the most generous situation--Any linear system of equations can be applied as long as they are produced by using FDM or FEM discretization; more importantly, it has the best parallel properties--this algorithm is in natural parallel.
A Domain Decomposition Solver for Acoustic Scattering by Elastic Objects in Layered Media
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Abstract: Finite element solution procedure is presented for accurately computing time-harmonic acoustic scattering by elastic targets buried in sediment. An improved finite element discretization based on trilinear basis functions leading to fourth-order phase accuracy is described. For sufficiently accurate discretizations 100 million to 1 billion unknowns are required. The resulting systems of linear equations are solved iteratively using the GMRES method with a domain decomposition preconditioner employing a fast direct solver. Due to the construction of the discretization and preconditioner, iterations can be reduced onto a sparse subspace associated with the interfaces. Numerical experiments demonstrate capability to evaluate the scattered field with hundreds of wavelengths.

Spline Solution of Generalized Un-damped Sine-Gordon Equation
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Abstract: The nonlinear un-damped Sine-Gordon equation is used to model many nonlinear phenomena. Numerical simulation of the solution of one-dimensional generalized sine-Gordon equation is considered. Two implicit three time-level spline difference schemes are developed for the numerical solution of one-dimensional Sine-Gordon equation, based on spline function approximation. The local truncation error, stability and convergence analysis of the resulting spline difference schemes are discussed in detailed. The arising linear system can be solved by LU Decomposition method. In the end, a numerical example is provided to demonstrate the effectiveness of the proposed schemes.
A Neumann-Neumann Domain Decomposition Algorithm For Tresca Problem

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Abstract: Recently, a co-called Neumann-Dirichlet algorithm for the solution to the contact problem with friction has been proposed and studied in the continuous and discretized setting in [1, 2]. This algorithm consists to solve in each iteration a linear Neumann problem for one body and a non-linear contact problem with friction for the other by using essentially the contact interface as the boundary data transfer.

In this work, we propose a Neumann-Neumann domain decomposition algorithm to approximate a contact problem between two elastic bodies with given friction and we prove its convergence. The Neumann-Neumann algorithm is the parallel one, in which we have to solve a Dirichlet problem and then a Neumann one, simultaneously on each domain. We also present some numerical results asserting the efficiency of this algorithm.

References


Multi-level Domain Decomposition Preconditioners for Simulating Flow in Complex Geological Media

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Abstract: We focus on flow in complex geological media, with spatial variability on multiple scales. Modeling challenges from porous media flow include strong coupling between the scales. For accurate representation of fluid flow, the fine scale variability is captured with residual free basis functions. While most multi-scale methods have only been studied for two levels, we explore the extension to three
and more scales. Mass conservation is an important property for fluid flow simulation, and is essential for correct representation of flow. Our multi-scale methods are applied also as multi-level preconditioners, where we require mass conservation on each level. We examine the efficiency of these preconditioners when applied on different grid structures. We will pay particular interest in the properties of the resulting flow field as they relate to fine-scale simulations.
Uzawa Domain Decomposition Method for the Stokes Problem

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Abstract: The Stokes problem plays an important role in computational fluid dynamics since it is encountered in the time discretization of (incompressible) Navier-Stokes equations by operator-splitting methods. Space discretization of the Stokes problem leads to large scale ill-conditioned systems. The Uzawa (preconditioned) conjugate gradient method is an efficient method for solving the Stokes problem. The Uzawa conjugate gradient method is a decomposition coordination method with coordination by a Lagrange multiplier.

We propose an additional (interface) continuity condition in the constrained minimization formulation of the Stokes problem. We then derive a decomposition coordination method with two multiplier: the pressure (for the divergence free condition) and the interface multiplier (for the continuity condition). At each step of our domain decomposition algorithm, we solve uncoupled scalar Poisson sub-problem. We study preconditioning by the Steklov-Poincaré operator.

A Simple Projection Method for the Solution of Discrete Thin PlateSplines

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Abstract: Data fitting is an integral part of a number of applications including data mining, 3D reconstruction of geometric models, image warping and medical image analysis. A commonly used method for fitting functions to data is the thin-plate spline method. This method is popular because it is not sensitive to noise in the data.

Traditional thin plate splines use radial basis functions that produce dense linear system of equations whose size increases with the number of data points. This limits the use of such techniques.

In a previous DD conference, DD17, we presented a discrete thin-plate spline method that uses piecewise functions with local support defined on a finite element mesh. The advantage of using functions with local support is that the dimension of the resulting system of sparse equations depends only on the number of grid points in the finite element mesh, not the number of data points.

Another advantage is that an iterative solver, such as the conjugate gradient method, can be used to solve the system. However it can be shown that the system of equations are similar to those arising from
Tikhonov regularisation, and consequently the equations are ill-conditioned for certain choices of the smoothing parameter.

We recently formulated a simple preconditioning technique based on the Sherman-Morrison-Woodbury formula. The formula allows us to divide the domain up into regions where there are lots of data points, and the interpolant is well defined, and regions where there are few data points. By using different types of grids in each region we are able to solve the problem for a much wider range of parameters. In the talk we plan to present this new technique and verify its effectiveness by applying it to some example data sets.
Numerical Method for Antenna Radiation Problem by FDTD Method with PML

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Abstract: In the numerical simulation of electromagnetic wave radiation problem from an antenna, the antenna region is assumed to be a perfectly conducting obstacle. We show numerically that it can be modeled effectively by a highly conducting region. The Finite Difference Time Domain method combined with Perfectly Matched Layer gives a flexible numerical methodology for this problem. We apply the method to several radiation problems with different types of antennas such as birdcage and Yagi where the delta gap type power supply model is adopted. For treating the unbounded outer region, we apply the newly developed technique to discretize the PML region with little artificial reflection whose theoretical justification in 1D case has been done previously in DD17 conference, and it is also effective in 2D and 3D cases at least numerically. We observed good 3D numerical performance of the method and confirmed its validity whose theoretical justification is still on going.

A Jacobian-free Implicit Immersed Interface Method for Stokes Flows Involving Moving Interfaces

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Abstract: In this work, a finite difference MAC scheme is presented for solving the steady Stokes equations with moving interfaces and the Dirichlet boundary condition. The moving interfaces are represented by Lagrangian control points and the location of the interfaces is updated implicitly using a Jacobian-free approach within each time step. The forces at the moving interfaces are calculated from the configuration of the interfaces and interpolated using cubic splines and then applied to the fluid through the related jump conditions. The proposed Jacobian-free Newton-GMRES method avoids the need to form and store the matrix explicitly in the computation of the inverse of the Jacobian and betters numerical stability. The Stokes equations are discretized on a MAC grid via a second-order finite difference scheme with the incorporation of jump contributions and the resulting saddle point system is solved by the conjugate gradient Uzawa-type method. Numerical results demonstrate very well the accuracy and effectiveness of the proposed method.
Semismooth Newton-Krylov-Schwarz Algorithms for Nonlinear Complementarity Problems

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Abstract: We present a semismooth Newton-Krylov-Schwarz algorithm for some nonlinear complementarity problems including obstacle problems and free boundary value problems. The semismooth Newton-Krylov-Schwarz algorithm consists of an inexact semi-smooth Newton, a Krylov subspace linear solver and a one-level or two-level overlapping restricted Schwarz preconditioner. We show numerically that such an approach is totally scalable in the sense that the number of Newton iterations and the number of linear iterations are both independent of the mesh size and the number of processors. In addition, the method is not sensitive to the sharp discontinuity often associated with obstacle problems. We present numerical results for several large scale calculations obtained on machines with hundreds of processors.

Conservative Parallel Schemes for Diffusion Equations

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Abstract: Many kinds of unconditional stable parallel discrete scheme for the diffusion problem have been constructed, but they are not conservative due to the flux being discontinuous across the inner interface between sub-domains. On the other hand, all the known conservative parallel schemes are conditional stable. No one parallel difference scheme can satisfy all the following requirements: (i) conservation; (ii) unconditional stability; (iii) second order convergence. We have constructed some parallel schemes which satisfy all of the requirements, moreover they can be easily implemented on the parallel computer, and have high degree of parallelism. Numerical results demonstrate the performance of our conservative parallel schemes.

A Multi-level Jacobian-free Method for 2D Four-field Extended MHD with Optimized Adaptive Meshes

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Abstract: A multi-level Jacobian-free algorithm is introduced for solving nonlinear partial differential equations. The 2D four-field extended MHD equation set (the "Harris Reconnection" problem) is derived from the a set of 3D basic equations describing incompressible, two-fluid (electron and ion), quasi-neutral plasma. This nonlinear problem is solved with fully implicit, Jacobian-free, multigrid method with a right preconditioning. Moreover, an optimized adaptive mesh generation technique is coupled into the system to resolve the requirements of grid resolution near the singularity point in the physical domain.